

Clarifications to Questions and Criticisms on the Johansen-Ledoit-Sornette Bubble Model

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The Johansen-Ledoit-Sornette (JLS) model of rational expectation bubbles with finite-time singular crash hazard rates has been developed to describe the dynamics of financial bubbles and crashes. It has been applied successfully to a large variety of financial bubbles in many different markets. Having been developed for more than one decade, the JLS model has been studied, analyzed, used and criticized by several researchers. Much of this discussion is helpful for advancing the research. However, several serious misconceptions seem to be present within this collective conversation both on theoretical and empirical aspects. Several of these problems appear to stem from the fast evolution of the literature on the JLS model and related works. In the hope of removing possible misunderstanding and of catalyzing useful future developments, we summarize these common questions and criticisms concerning the JLS model and offer a synthesis of the existing state-of-the-art and best-practice advices.

Keywords: JLS model, financial bubbles, crashes, log-periodic power law, fit method, sloppiness, taboo search, bootstrap, probabilistic forecast.

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I. INTRODUCTION

The Johansen-Ledoit-Sornette (JLS) model [1–4] has been developed to describe the dynamics of financial bubbles and crashes. The model states that bubbles are not characterized by exponential increase of price but rather by faster-than-exponential growth of price. This phenomenon is generated by behaviors of investors and traders that create positive feedback in the valuation of assets and unsustainable growth, leading to a finite-time singularity at some future time t_c . From a technical view point, the positive feedback mechanisms include (i) option hedging, (ii) insurance portfolio strategies, (iii) market makers bid-ask spread in response to past volatility, (iv) learning of business networks and human capital build-up, (v) procyclical financing of firms by banks (boom vs contracting times), (vi) trend following investment strategies, (vii) asymmetric information on hedging strategies viii) the interplay of mark-to-market accounting and regulatory capital requirements. From a behavior view point, positive feedbacks emerge as a result of the propensity of humans to imitate, their social gregariousness and the resulting herding. This critical time t_c of the model is interpreted as the end of the bubble, which is often but not necessarily the time when a crash occurs in the actual system. During this growth phase, the tension and competition between the value investors and the noise traders create deviations around the power law growth in the form of oscillations that are periodic in the logarithm of the time to t_c . Combining these two effects, this model succinctly describes the price during a bubble phase as log-periodic power law (LPPL).

Over longer than the past decade, the JLS model has been used widely to detect bubbles and crashes ex-ante (i.e., with advanced documented notice in real time) in various kinds of markets such as the 2006-2008 oil bubble [5], the Chinese index bubble in 2009 [6], the real estate market in Las Vegas [7], the U.K. and U.S. real estate bubbles [8, 9], the Nikkei index anti-bubble in 1990-1998 [10] and the S&P 500 index anti-bubble in 2000-2003 [11]. Other recent ex-post studies include the Dow Jones Industrial Average historical bubbles [12], the corporate bond spreads [13], the Polish stock market bubble [14], the western stock markets [15], the Brazilian real (R\$) - US dollar (USD) exchange rate [16], the 2000-2010 world major stock indices [17], the South African stock market bubble [18] and the US repurchase agreements market [19]. Moreover, new experiments in ex-ante bubble detection and forecast has been launched since November 2009 in the Financial Crisis Observatory at

ETH Zurich [20–22].

Having been developed for more than one decade, the JLS model has been studied, used and criticized by many researchers including Feigenbaum [23], Chang and Feigenbaum [24, 25], van Bothmer and Meister [26], Fry [27], and Fantazzini and Geraskin [28]. The most recent papers addressing the pros and cons of past works on the JLS model are written by Bree and his collaborators [29, 30]. Many ideas in these last two papers are correct, pointing out that some of the earlier works had some inconsistencies. However, there are some serious misunderstandings present of both the theoretical and empirical parts of the model. Therefore, it is necessary to address and clarify the misconceptions that some researchers seem to hold concerning this model and to provide an updated, concise reference on the JLS model.

The structure of this paper is as follows. Section II discusses the questions about the theory and derivation of the JLS model. The questions on fitting methods of the model are commented in Section III. Issues on probabilistic forecast will be addressed in Section IV. We conclude in Section V.

II. DISCUSSIONS ON THEORY OF THE JLS MODEL

We will give the derivation of the JLS model first in this section. Then we discuss three issues related to the derivation and the proper parameter ranges.

A. Derivation of the JLS model

The derivation of the JLS model is quite transparent. It starts from assuming the dynamics of the price satisfy a simple stochastic differential equation with drift and jump:

$$\frac{dp}{p} = \mu dt + \sigma dW - \kappa dj, \quad (1)$$

where p is the stock market price, μ is the drift (or trend) and dW is the increment of a Wiener process (with zero mean and unit variance). The term dj represents a discontinuous jump such that $j = 0$ before the crash and $j = 1$ after the crash occurs. The loss amplitude associated with the occurrence of a crash is determined by the parameter κ . Each successive crash corresponds to a jump of j by one unit. The dynamics of the jumps is governed by a

crash hazard rate $h(t)$. Since $h(t)dt$ is the probability that the crash occurs between t and $t + dt$ conditional on the fact that it has not yet happened, we have $E_t[dj] = 1 \times h(t)dt + 0 \times (1 - h(t)dt)$ and therefore

$$E_t[dj] = h(t)dt. \quad (2)$$

Under the assumption of the JLS model, noise traders exhibit collective herding behaviors that may destabilize the market. The model assumes that the aggregate effect of noise traders can be accounted for by the following dynamics of the crash hazard rate:

$$h(t) = B'(t_c - t)^{m-1} + C'(t_c - t)^{m-1} \cos(\omega \ln(t_c - t) - \phi'). \quad (3)$$

The cosine part of the second term in the r.h.s. of (3) takes into account the existence of possible hierarchical cascades of accelerating panic punctuating the growth of the bubble, resulting from a preexisting hierarchy in noise trader sizes and/or the interplay between market price impact inertia and nonlinear fundamental value investing. The non-arbitrage condition reads $E_t[dp] = 0$, which leads to $\mu(t) = \kappa h(t)$, by taking the expectation of (1) with the condition that no crash has yet occurred. Using this and substituting (3) and integrating yields the so-called log-periodic power law (LPPL) equation:

$$\ln E[p(t)] = A + B(t_c - t)^m + C(t_c - t)^m \cos(\omega \ln(t_c - t) - \phi) \quad (4)$$

where $B = -\kappa B'/m$ and $C = -\kappa C'/\sqrt{m^2 + \omega^2}$. Note that this expression (4) describes the average price dynamics only up to the end of the bubble. The JLS model does not specify what happens beyond t_c . This critical t_c is the termination of the bubble regime and the transition time to another regime.

B. Why m should be between 0 and 1?

We claim that the parameter m in the JLS model should be between 0 and 1. Bree and Joseph asked why m cannot be greater than 1 in [29]. The answers are:

1. For $m < 1$, the crash hazard rate accelerates up to t_c but its integral up to t , which controls the total probability for a crash to occur up to t , remains finite and less than 1 for all times $t \leq t_c$. It is this property that makes rational for investors to remain invested knowing that a bubble is developing and that a crash is looming. Indeed, there is still a finite probability that no crash will occur during the lifetime of the bubble. The excess return

$\mu(t) = \kappa h(t)$ is the remuneration that investors require to remain invested in the bubbly asset, which is exposed to a crash risk. The crash hazard may diverge as t approaches a critical time t_c , corresponding to the end of the bubble.

2. Within the JLS framework, a bubble exists when the crash hazard rate accelerates with time. According to (3), this imposes $m < 1$ and $B' > 0$. That is, $m \geq 1$ cannot lead to an accelerating hazard rate.

3. Finally, the condition that the price remains finite at all time, including t_c , imposes that $m > 0$.

Summarize the points above, we conclude that a proper range of m where the bubble occurs should be $0 < m < 1$.

C. Non-negative risk condition

van Bothmer and Meister derived a constraint on the variables of the JLS model [26] from the statement that the crash rate should be non-negative. It states that:

$$b := -Bm - |C|\sqrt{m^2 + \omega^2} \geq 0. \quad (5)$$

Most current research using the JLS model has taken this restriction into consideration. It is among the basic restrictive filters for identifying bubbles in a more modern framework. In [31–34], b in (5) is even used as a key parameter for pattern recognition method to detect the market rebounds.

D. How can the price in the JLS model be decreasing during a bubble?

Bree and Joseph claim that “the mechanism proposed to lead to LPPL fluctuations as reported in [2] must be incorrect as it requires the price to be increasing throughout the bubble.” Bree and Joseph are completely mistaken here. Section II A presents the JLS model in a self-consistent way. The error of Bree and Joseph is that they do not realize that, even if not specified, the definition of the JLS model includes implicitly the stochastic term σdW as in expression (1). In expectations, this term disappears, hence it is not included in the description of the initial JLS paper. But, Bree and Joseph are wrong to conclude that the JLS model imposes that the price is always monotonously increasing. Note that the

formulation is nothing but that of the rational expectation of [35], which follows exactly the same procedure, with a stochastic component which does not play a role in the specification of the crash hazard rate relationship to the μ term, but is present to ensure that the price can indeed decrease. As indicated at the section II C, note that van Bothmer and Meister showed that a certain condition between the parameters of the LPPL fit should hold in order for the crash hazard rate to remain positive at all times until the end of the bubble, *but* this condition is *not* that the price should be non-decreasing or always increasing!

E. Faster-than-exponential growth in the JLS model

One of the fundamental differences between the JLS model and standard models of financial bubbles is that the JLS model claims that the price follows a faster-than-exponential growth rate during the bubble. It is necessary to emphasize this statement as many researchers make mistakes here. For example, Bree and Joseph wrote “exponential growth is posited in the LPPL” in several places in [29].

Financial bubbles are generally defined as transient upward acceleration of prices above the fundamental value [36–38]. However, identifying unambiguously the presence of a bubble remains an unsolved problem in standard econometric and financial economic approaches [39, 40], due to (i) the fact that the fundamental value is in general poorly constrained and (ii) the difficulty in distinguishing between an exponentially growing fundamental price and exponentially growing bubble price. As we have already described, the JLS model defines a bubble in terms of faster-than-exponential growth [41]. Thus, the main difference with standard bubble models is that the underlying price process is considered to be intrinsically transient due to positive feedback mechanisms that create an unsustainable regime. See for instance [6] where this is made as clear as possible.

F. Which kind of bubbles can be detected by the JLS model?

In page 4 of [29], three claims are outlined. One of them states that: “Financial crashes are preceded by bubbles with fluctuations. Both the bubble and the crash can be captured by the LPPL when specific bounds are imposed on the critical parameters β and ω ”, where β is presented as m in this paper.

Here, we should stress that this above claim is not entirely correct because crashes can be endogenous or exogenous. The JLS model is suitable only for endogenous crashes! Or more precisely, the JLS model is for bubbles, not for crashes. Endogenous crashes are preceded by the bubbles that are generated by positive feedback mechanisms of which imitation and herding of the noise traders are probably the dominant ones among the many positive feedback mechanisms inherent to financial system. In the abstract of the reference [41], Johansen and Sornette state: “Globally over all the markets analyzed, we identify 49 outliers (now referred more appropriately as “dragon-kings” [42]), of which 25 are classified as endogenous, 22 as exogenous and 2 as associated with the Japanese anti-bubble. Restricting to the world market indices, we find 31 outliers, of which 19 are endogenous, 10 are exogenous and 2 are associated with the Japanese anti-bubble.” Although the endogenous outliers are more frequent than the exogenous ones, the exogenous outliers still constitute a quite large portion. Therefore, the JLS model cannot capture all of the crashes in the market. Only endogenous crashes which are preceded by the bubbles can be captured by the JLS model.

III. FITTING PROBLEMS CONCERNING THE JLS MODEL

A. Extensions of the JLS model and their calibration

The form of the JLS model we obtained in (4) is called the first-order LPPL Landau JLS model. Extensions have been proposed, essentially amounting to choosing alternative forms of the crash hazard rate $h(t)$ that replace expression (3). Let us mention the so-called second-order and third-order LPPL Landau models [10, 11, 43–45], the Weierstrass-type LPPL model [46, 47], the JLS model extended with second-order and third-order harmonics [7, 18, 48, 49] and the JLS-factor model in which the LPPL bubble component is augmented by other financial risks factors [50, 51]. We should also mention that a non-parametric estimation of the log-periodic power law structure has been developed to complement the above parametric calibrations [52]. These extensions are warranted by the fact that the positive feedback mechanisms together with the presence of the symmetry of discrete scale invariance can be embodied in a general renormalization group equation [46], whose general solution is the Weierstrass-type LPPL model. Then, the first-order LPPL Landau JLS model can be considered to be just the first term in a general log-periodic Fourier series expansion

of the general solution. Therefore, further away from the critical time t_c , corrections from the first-order expression can be expected to be relevant, depending on the context. In addition, nonlinear extensions to the renormalization group are embodied partially in the second-order and third-order LPPL Landau models, which extend the time domain over which the model can be calibrated to the empirical data [53].

Let us mention that Sornette and Johansen [53] discussed the difference between the fitting results obtained using the first order and the second order LPPL Landau-type JLS models. They used daily prices of the S&P 500 index from 1980 to 1987. The results show that the fitting result of the second order form is much better than the first order form, as based on the measure of residual sum of squares. A standard Wilks test of nested hypotheses confirms the fact that the second-order form provides a statistically significant improvement over the first-order form (recall that the first-order LPPL Landau formula is recovered as a special case of the second-order LPPL Landau formula, hence the first model is nested within the second model). We reproduce the fitting results from [53] in Fig. 1 to give an intuition on the difference between the first order and second order LPPL Landau fits. One can observe that the first-order LPPL Landau formula accounts reasonably well for the data from mid-1985 to the peak in October 1987. In contrast, the second-order LPPL Landau formula provides a good fit all the way back to the beginning of 1980. This result helps explain why the results quoted by Bree et al. [30] for time windows of 834 trading days may be questionable.

Notwithstanding the improvement provided by the second-order LPPL Landau model for large time windows, it is sufficient in many cases to use the first-order version just to get a diagnostic of the presence of a bubble. This is true even when the time window is larger than 2-3 years. For instance, the first-order LPPL Landau model was implemented within a pattern recognition method [31, 32, 34] with time windows of up to 1500 days. The key to the reported performance in forecasting [31, 32, 34] is the combination of bubble diagnostics at multiple time scales, with common model parameters associated with robustness.

B. Selection of the start of the time window

A common question arising in fitting the JLS model is to decide which date t_1 should be selected as the beginning of the fitting time window. Bree and Joseph [29] are more

consistent in defining bubbles to analyze and their exact beginnings than the papers of 1998-2000 they analyze, which date from a decade at least. This is a very good approach. However, we would like to note that more recent procedures are more systematic as shown for instance in [6, 20, 28, 32, 33, 54]. In these more recent procedures, multiple t_1 's are selected to make the prediction more statistically reliable. The key point here is that a single t_1 — corresponding to a single fit window — is unreliable and an ensemble of fits should be used.

C. Should real price or log-price be fitted?

As recalled by Bree and Joseph in [29], Sornette and Johansen [1] argued that log-price should be used when the amplitude of the expected crash is proportional to the price increase during the bubble. This is because (4) is derived from (1), which assumes that the changing price dp is proportional to the price p . Therefore, this statement is in accordance with Bree et al.'s definition of a crash (25% drop in price) in [29, 30]. Hence, it seems that the attempt by Bree et al. to compare the results of the fits when using the price (and not the log-price) is inconsistent.

One can also investigate the possibility that price changes may not be proportional to price. If this is the case, use of the real price is warranted according to the arguments put forward by Sornette and Johansen [1]. In practice, it is useful to try both fitting procedures with prices and log-prices and compare their relative merits. But one should be cautious because the fits using prices (and not log-prices) involve data values that may change over several orders of magnitude over the time window of interest. As a consequence, the standard least square fits is not suitable anymore. Instead, a normalized least square minimization is recommended so that each data point of the time series roughly contributes equally to the mean-square root diagnostic. This approach has been implemented recently in Ref. [54].

D. Sloppiness of the JLS models and search algorithm

Bree et al. [29, 30] claim that the concept of sloppiness and its consequence should be considered in fitting the JLS model to some empirical data with the Levenberg-Marquart algorithm. And they challenge the relevance of the obtained fits. It seems to us that

this claim overlooks that the correct fitting procedure should include the combination of the Levenberg-Marquart algorithm [55] *and* a preliminary taboo search [56] or other meta-heuristics such as genetic algorithm and simulated annealing algorithm. This should occur together with the slaving of the linear parameters to the non-linear ones in order to reduce the effective number of parameters from 7 to 4 (and to 3 in the recently novel procedure of Filimonov and Sornette [57]). The taboo search is a very good algorithm that provides a robust preliminary systematic exploration of the space of solutions, which prevents the Levenberg-Marquart algorithm later on to be stuck in special regions of the space of solutions. Also in a standard fitting procedure, the many results that may be obtained from the taboo search (i.e. results associated with different parts of the searching space) should be kept.

Taking into account the two points mentioned above, the quality of the fits with the JLS model is in general adequate [5–7, 18, 19, 54]. In contrast, it is obvious that fits using only the Levenberg-Marquart algorithm without a reasonably initial guess and sufficient preliminary exploration of the space of solutions will produce spurious results, with most of the parameters stuck at the boundary of the search space. A typical example of such fitting failure is shown in Ref.[30], where all the fitted m values are either very close to 0 or close to 1 and almost all the fitted t_c and ω values are very close to 0.

Reference [29] provides a sensitivity analysis of the root mean square error (RMSE), in which one parameter is scanned while the others remain fixed. The problem is that, because of the nonlinearity of expression (4), it is not obvious that the results of such a scan can be trusted. That is, if local minima in, say, ω are found while the other parameters are kept fixed, do the same minima appear when one or more of the other parameters are changed to different values? In other words, is the multi-dimensional parameter landscape around these minima smooth? The answer to this question is more important than showing the sensitivity of 2 dimensional subspaces, as in [29]. In practice, answering this question on the smoothness of the multi-dimensional parameter landscape is difficult. Filimonov and Sornette [57]) have documented that the standard slaving of three linear parameters (A, B and C in expression (4)) to the four remaining nonlinear parameters results into a quite corrugated fitness landscape that requires meta-heuristic (such as the taboo search). The meta-heuristic simultaneously changes all parameters to find acceptable minima as starting points for the Levenberg-Marquart algorithm. Yes, this approach does not guarantee finding *the* absolute minimum but it does provide *an ensemble of acceptable local minima*. This

ensemble approach is more robust than searching in vain for a single global minimum.

E. Performance of the recommended fitting method on synthetic data

It is an essential building stone of any fitting procedure that it should be tested on synthetic data. Indeed, in any calibration exercise, one faces simultaneously two unknown: (i) the performance, reliability and robustness of the calibration procedure and (ii) the time series under study from which one hopes to extract meaningful information. How can one learn about an unknown dataset if one does not fully understand how the fitting method behaves on controlled well-known data sets? Early on, Johansen et al. [2, 3] set the stage by developing comparative tests on synthetic time series generated by the GARCH model. We also attract the attention to the fact that one of the most extensive set of synthetic tests concerning the possible existence of spurious log-periodicity is found in reference [58]. Zhou and Sornette [59] presented a systematic study of the confidence levels for log-periodicity only, using synthetic time series with many different types of noises, including noises whose amplitudes are distributed according to power law distributions with different exponents and long-memory modeled by fractional Brownian noises with various Hurst exponents spanning the full range from anti-persistent ($0 < H < 1/2$) to persistent ($1/2 < H < 1$).

We now show that the current fitting methods estimate the parameters of the JLS model within a reasonable range of uncertainty in the following. For this, a reference log-periodic power law (LPPL) time series of duration equal to 240 days is generated for a typical set of parameters, shown in Table I. This series corresponds to a value of the critical time t_c equal to 300 (days). The choice of 240 days for the time window size is motivated by the typical length for the generation of bubbles found in various case studies in the literature.

The synthetic data is generated by combining the LPPL time series with noise. Two kinds of noise are considered: Gaussian noise and noise generated with a Student t distribution with four degrees of freedom (which exhibits a tail similar to that often reported in the literature for the distribution of financial returns). For both types of noise, the mean value is zero and the standard deviation is set to be 5% of the largest log-price among the 240 observations in the reference series. The standard deviation is chosen quite high in order to offer stringent test of the efficiency of the current fitting method. Synthetic samples obtained with both types of noise along with the reference time series are shown in Fig. 2.

	Reference	Mean (std) of Gaussian	Mean (std) of Student's t
t_c	300	296.07 (20.44)	295.15 (20.81)
m	0.7	0.74 (0.15)	0.72 (0.18)
ω	10	9.75 (1.43)	9.71 (1.47)

TABLE I: The parameter values used to generate the synthetic data are shown in the second column “Reference”. The mean and standard deviation values of the parameters obtained by fitting the JLS model to the synthetic LPPL time series decorated by the two types of noise discussed in the text are given in the last two columns. These numbers are estimated from 200 statistical realizations of the noise, and each realization is characterized by ten different best fits with the Levenberg-Marquart algorithm, leading to a total of 2000 estimated parameters. The other parameters used to generate the synthetic LPPL are $\phi = 1, A = 10, B = -0.1, C = 0.02$.

For each type of noise, 200 synthetic time series are generated. We fit each series with the JLS model (4) and keep the ten best fits for each one. Recall that our stochastic fit method produces multiple ‘good’ fits instead of the ‘best’ fit, which, in practice, is difficult, if not impossible, to find. In the new procedure developed by Filimonov and Sornette recently [57], the ‘best’ fit can be found in most cases that are qualified to be in a bubble regime. However, we still use the standard heuristic procedure in the present paper. This best ten selection results in 2000 sets of estimated parameters for each type of noise. The probability density functions of t_c , m and ω for the two types of noise are calculated by a non-parametric method (adaptive kernel technique). The results are shown in Fig. 3.

The mean and standard deviation of these parameters are shown in Table I alongside the original numbers used to generate the true LPPL function ($t_c = 300, m = 0.7, \omega = 10$) without noise. This test on synthetic data demonstrates that the fitting method combining the meta-heuristic Taboo search with the Levenberg-Marquart algorithm is satisfactory. We observe negligible biases, especially for the crucial critical time parameter t_c . The standard deviation for t_c of about 20 days is three times smaller than the 60 days separating the last observation (day 240) of the time series and the true critical time occurring at the 300-th day, showing that the calibration of a time series exhibiting LPPL structure, even with very large statistical noise, can provide significant skills in forecasting the critical time t_c .

IV. PROBABILISTIC FORECAST

From a practical risk management view point, one of the prizes obtained from the calibration of the JLS model to financial time series is the estimation of the most probable time of the end of the bubble t_c , which can take the form of a crash, but is more generally a smooth transition to a new market regime.

As we mentioned before, a distribution of t_c is obtained for a single bubble period, associated with the set of fitted time windows (see Section IIIB) and the recording of multiple locally optimal fits from the stochastic taboo search (see Section IIID). Recall that the output of the meta-heuristic is used as the initial guess required by the Levenberg-Marquart algorithm. As demonstrated in the previous subsection, the estimation of the distribution of the most probable time t_c for the end of the bubble is generated by a reliable non-parametric method [60].

Bree et al. [30] make the interesting remark that the estimation of the probability density of t_c might be improved by augmenting the analysis of the original time series with that of many replicas. These replicas of the initial time series can be obtained for instance by using a LPPL function obtained for the first calibration on the original time series and adding to it noise generated by an AR(1) process. This methodology provides a measure of robustness of the whole estimation exercise. The choice of an AR(1) process for the noise is supported by the evidence provided in Refs. [61, 62] that the residuals of the calibration of the JLS model to a bubble price time series can be reasonably described by an AR(1) process. But, this is only one among several possibilities. Another one, that we have implemented in our group for quite some time and now use systematically, is to generate bootstraps in which the residuals of the first calibration on the original time series are used to seed as many synthetic time series as needed, using reshuffled blocks of residuals of different durations. For instance, reshuffling residuals in blocks of 25 days ensures that the dependence structure between the residuals is identical in the synthetic time series as in the original one up to a month time scale. Note that this bootstrap method does not assume Gaussian residuals in contrast with the AR(1) noise generation model. It captures also arguably better the dependence structure of the genuine residuals than the linear correlation embedded in the AR(1) model.

V. CONCLUSION

We have discussed the present theoretical status and some calibration issues concerning the Johansen-Ledoit-Sornette (JLS) model of rational expectation bubbles with finite-time singular crash hazard rates. We have provided a guide to the advances that have punctuated the development of tests of the JLS model performed on a variety of financial markets during the last decade. We can say that the development of new versions and of methodological improvements have paralleled the occurrence of several major market crises, which have served as inspirations and catalyzers of the research. We believe that the field of financial bubble diagnostic is progressively maturing and we foresee a close future when it could become operational to help decision makers alleviate the consequences of excess leverage leading to severe market dysfunctions.

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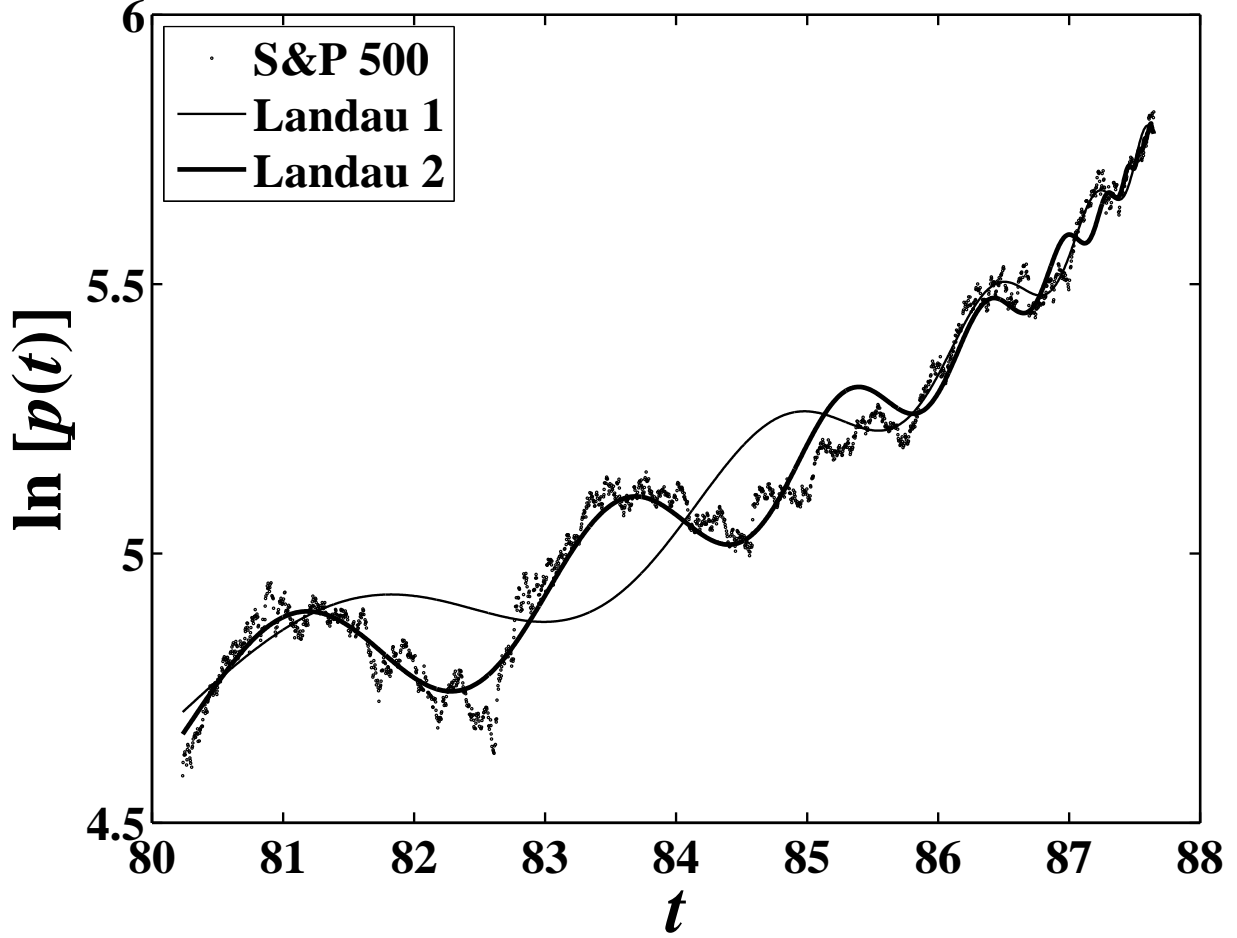


FIG. 1: Time dependence of the logarithm of the New York stock exchange index S&P 500 from January 1980 to September 1987 and the best fits by the first and the second order LPPL Landau models. The crash of October 14, 1987 corresponds to 1987.78 decimal years. The thin line represents the best fit with the first-order LPPL Landau model on the interval from July 1985 to the end of September 1987 and is shown on the whole time span since January 1980. The thick line is the fit by the second-order LPPL Landau model from January 1980 to September 1987. (Reproduced from [53])

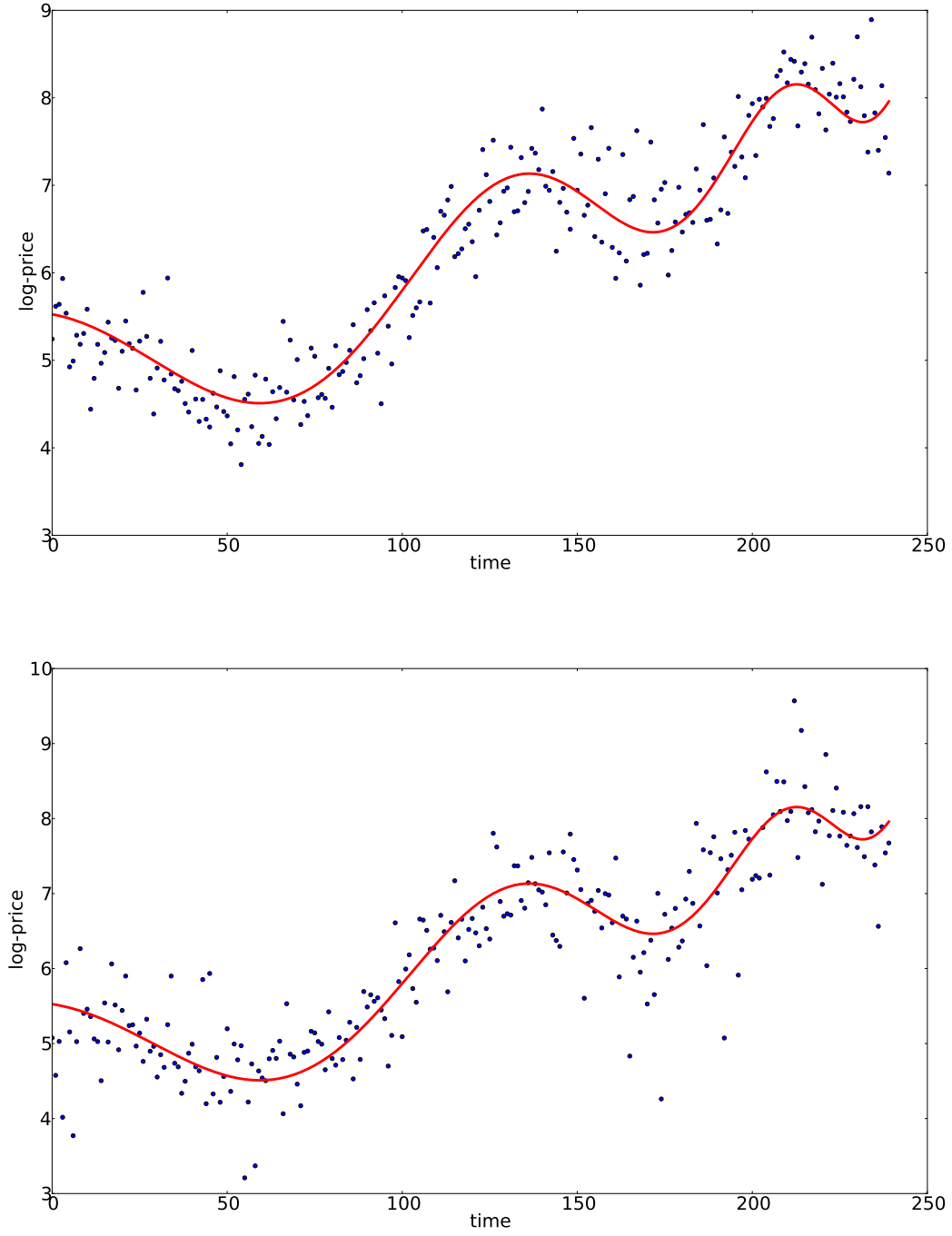


FIG. 2: Synthetic data examples with zero mean and large standard deviation (5% of the largest log-price among 240 reference points). Upper panel: the synthetic data with Gaussian noise. Lower panel: the synthetic data with noise generated with a Student t distribution with four degrees of freedom. The red solid line shows the reference LPPL time series.

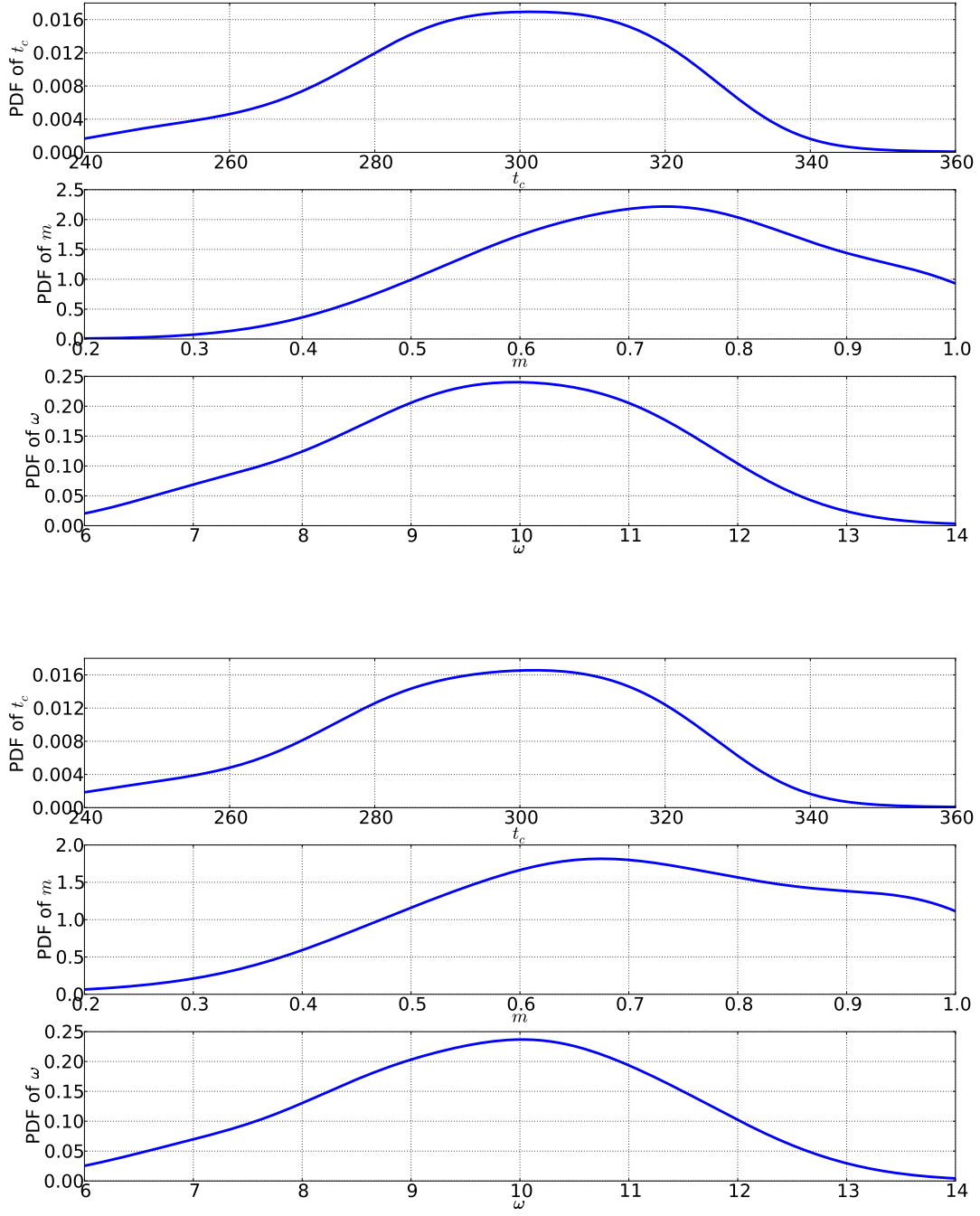


FIG. 3: Probability density functions of t_c , m and ω obtained by a non-parametric kernel method applied to the parameter values determined by analyzing 200 synthetic time series, each of which being characterized by its ten best fits with the Levenberg-Marquart algorithm, leading to a total of 2000 estimated parameters.